Special Series →

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1)

$$n \cdot 1 + (n-1) \cdot 2 + (n-2) \cdot 3 + \dots\dots + \overbrace{2 \cdot (n-1)} + 1 \cdot n$$

~~4+1+3+2+2+3~~ 14

No. of terms $= n$ (Fixed)

General term $a_r = (n - (r-1)) \cdot r$

$$= (n - r + 1) \cdot r$$

$$S_n = \sum_{r=1}^{n} a_r = \sum_{r=1}^{n} (n-r+1) \cdot r$$

$$\sum_{r=1}^{n} (n+1) r - r^2$$

$$(n+1) \sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2$$

$$(n+1) \left\{ \frac{n(n+1)}{2} \right\} - \left\{ \frac{n(n+1)(2n+1)}{6} \right\}$$

$$\frac{n(n+1)}{2} \left\{ (n+1) - \left(\frac{2n+1}{3} \right) \right\}$$

$$\frac{n(n+1)}{2} \left\{ \frac{3n + 3 - 2n + 1}{3} \right\}$$

$$S_n = \frac{n(n+1)(n+2)}{6}$$

Q2 →

1 — 1st row

2 3 — 2nd row

4 5 6

7 8 9 10

11 12 13 14 15 — 5th row

16 . . . . .

① Find sum of numbers in the n^{th} row

⇒ n^{th} row has n terms

$a = ??$ AP, $d = 1$

↓

First term of n^{th} row

1, 2, 4, 7, 11, ...

~~$S_n = 1+2+3+4$ ...... n terms~~

$S_n = 1 + 2 + 4 + 7 + 11 + 16 \dots\dots + a_n$

$~~S_n = 1+2+3+4~~ \ldots \qquad n-1 \text{ terms} \qquad ~~S_n~~ =$

$$S_n = 1 + 2 + 4 + 7 + 11 + 16 \ldots\ldots + a_n$$
$$S_n = \quad 1 + 2 + 4 + 7 + 11 \quad \cdots + a_{n-1} + a_n$$

$$0 = 1 + (1 + 2 + 3 + 4 \ldots\ldots\ldots + n-1) - a_n$$

$$0 = 1 + \frac{n(n-1)}{2} - a_n$$

$$a_n = \frac{2 + n^2 - n}{2} \Rightarrow \boxed{a_n = \frac{n^2-n+2}{2}}$$

Sum of the n^{th} row

$$\boxed{S_n = \frac{n}{2}(2a + (n-1)d)}$$

$$a = \frac{n^2-n+2}{2} \qquad d = 1 \qquad n = $$

$$S_n = \frac{n}{2}\left\{ 2\left(\frac{n^2-n+2}{2}\right) + (n-1)1 \right\}$$

$$S_n = \frac{n}{2}\left\{ n^2 \cancel{-n} + 2 + \cancel{n} - 1 \right\} = S_n = \frac{n}{2}\left\{ n^2 + 1 \right\}$$

$$\boxed{S_n = \frac{(n^2+1)n}{2}}$$

— x — x — x —

Q (ii) Find sum of numbers in all the n rows.

Last number of ~~every~~ n^{th} row...

$n-1$ terms

$$S_n = 1 + 3 + 6 + 10 + 15 + \ldots\ldots\ldots + a_n$$
$$S_n = \quad 1 + 3 + 6 + 10 \quad + a_{n-1} + a_n$$

$$0 = 1 + 2 + 3 + 4 + 5 + \ldots\ldots + n - a_n$$

$$\boxed{a_n = \frac{n(n+1)}{2}} \Rightarrow$$

Sum of all the rows...

$$\text{Sum} = 1 + 2 + 3 + \ldots\ldots\ldots + \frac{n(n+1)}{2}$$

$~~1+2+2+3~~$

$$\boxed{1 + 2 + \cdots + n = \frac{n(n+1)}{2}}$$

$$= \frac{\left(\frac{n(n+1)}{2}\right)\left(\frac{n(n+1)}{2} + 1\right)}{2}$$

$$\Rightarrow \frac{n(n+1)}{4}\left\{\frac{n^2+n+2}{2}\right\}$$

$$\Rightarrow \frac{n(n+1)(n^2+n+2)}{8}$$

HW

Q.

1

3 5

7 9 11

13 15 17 19

⋮ ⋮ ⋮ ⋮

Prove sum of numbers in the 'n'th row is n^3.

Prove

$$Q \rightarrow \frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \cdots\cdots\cdots + \underbrace{\frac{2n+1}{1^2+2^2+\cdots+n^2}}_{a_n} = \frac{6n}{n+1}$$

n terms

$6\left\{\frac{1}{n} - \frac{1}{n+1}\right\}$

3, 5, 7, ...

$a_n = 3 + (n-1)2$

$= 2n+1$

$$a_r = \frac{2r+1}{1^2+2^2+\cdots+r^2}$$

$$a_r = \frac{\cancel{2r+1}}{\frac{r(r+1)\cancel{(2r+1)}}{6}}$$

$$\boxed{a_r = \frac{6}{r(r+1)}}$$

$$a_r = \frac{6}{r(r+1)} = \frac{A}{r} + \frac{B}{r+1}$$

$$\Rightarrow 0r + 6 = A(r+1) + B(r)$$

Coef of 'r' $A+B=0$

Const coeff $A=6$

$B=-6$

$$\boxed{a_r = \frac{6}{r} + -\frac{6}{r+1}}$$

$$a_r = 6\left(\frac{1}{r} - \frac{1}{r+1}\right)$$

$S_n = \cancel{6}$

$$6\left(\frac{1}{1}-\frac{1}{2}\right)+6\left(\frac{1}{2}-\frac{1}{3}\right)+6\left(\frac{1}{3}-\frac{1}{4}\right)\ldots\ldots+6\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$6\left(1-\cancel{\frac{1}{2}}+\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}+\cancel{\frac{1}{3}}+\ldots\ldots\cancel{\frac{1}{n}}+\cancel{\frac{1}{n}}-\frac{1}{n+1}\right)$$

$$6\left(1-\frac{1}{n+1}\right)=6\left(\frac{n+1-1}{n+1}\right)=\boxed{\frac{6n}{n+1}}$$

Q →

$$\sum_{p=1}^{32}\left\{(3p+2)\left[\underbrace{\sum_{q=1}^{10}\left(\sin\left(\frac{2q\pi}{11}\right)-i\cos\frac{2q\pi}{11}\right)}\right]\right\}^{p}$$

$$\sum_{q=1}^{10}\left(\sin\left(\frac{2q\pi}{11}\right)-i\cos\left(\frac{2q\pi}{11}\right)\right)$$

$$\sum_{q=1}^{10}(-i)\left(\cos\left(\frac{2q\pi}{11}\right)+\frac{1}{-i}\sin\left(\frac{2q\pi}{11}\right)\right)$$

$$\sum_{q=1}^{10}(-i)\left(\cos\left(\frac{2q\pi}{11}\right)+i\sin\left(\frac{2q\pi}{11}\right)\right)$$

$$\sum_{q=1}^{10}(-i)\,e^{i\frac{2q\pi}{11}}$$

$$(-i)\left\{\sum_{q=1}^{10}e^{i\frac{2q\pi}{11}}\right\}$$

$$(-i)\left\{\underbrace{e^{i\frac{2\pi}{11}}+e^{i\frac{4\pi}{11}}+\ldots\ldots}_{10\text{ terms}}+e^{i\frac{20\pi}{11}}\right\}$$

GP

$$e^{\theta}+e^{2\theta}+\ldots\ldots+e^{n\theta}\Rightarrow\frac{e^{\theta}\left((e^{\theta})^{n}-1\right)}{(e^{\theta}-1)}$$

$a=e^{\theta}$

$r=e^{\theta}$

$i2\pi$

$$\sum_{x=1}^{4}\left\{(x^2-1)\sum_{y=1}^{3}y^{x}\right\}$$

$$\sum_{x=1}^{4}(x^2-1)(1^x+2^x+3^x)$$

$$(1^2-1)(1^1+2^1+3^1)+(2^2-1)(1^2+2^2+3^2)+\ldots\ldots+(4^2-1)(1^4+2^4+3^4)\ldots$$

Programing...

```
i=0;
for(i=0, i≤5, i++)
{ print i }
```

0, 1, 2, 3, 4, 5,

```
i=0;
j=0
for (i=0; i≤5, i++)
{
for (j=0; j≤3, j++)
{ print i,j }
}
```

0,0 1,0 2,0 3.... 5,0

0,1 1,1 2,1 5,1

$a = e^{\theta}$

$r = e^{\theta}$

$$\frac{e\left((e^{\theta})-1\right)}{(e^{\theta}-1)}$$

0,0	1,0	3,	2,0	3	5,0
0 1	~~1,1~~		2,1		5,1
0 2	~~1,2~~		2,2		5,2
0 3	1,3		2,3		5,3

$$(-i)\left\{\frac{e^{\frac{i2\pi}{\pi}} \times \left(\left(e^{\frac{i2\pi}{11}}\right)^{10}-1\right)}{\left(e^{\frac{i2\pi}{11}}-1\right)}\right\}$$

$$\frac{e^{i\theta}-1}{}$$

$$\Rightarrow \cos\theta + i\sin\theta - 1$$

$$(-i)\left\{\frac{e^{\frac{i2\pi}{11}}\left(e^{\frac{i20\pi}{11}}-1\right)}{\left(e^{\frac{i2\pi}{11}}-1\right)}\right\}$$

$$\Rightarrow (\cos\theta - 1) + i\sin\theta$$

$$(-i)\ e^{\frac{i2\pi}{11}}\frac{\left(\cancel{2i}\sin\frac{10\pi}{11}e^{\frac{i10\pi}{11}}\right)}{\left(\cancel{2i}\sin\frac{\pi}{11}\ e^{i\pi/11}\right)}$$

$$\Rightarrow -2\sin^2\theta/2 + i\,2\sin\theta/2\cos\theta/2$$

$$\Rightarrow 2\sin\theta/2\left(-\sin\theta/2 + i\cos\theta/2\right)$$

2nd quad.

$$\sin\left(\frac{10\pi}{11}\right) = \sin\left(\pi - \frac{\pi}{11}\right) \qquad \frac{e^{4x}}{e^{x}} = e^{4x-x} = e^{3x}$$

$$\Rightarrow 2i\sin\theta/2\left(\cos\theta/2 + i\sin\theta/2\right)$$

$$(= 2i\sin\theta/2\ e^{i\theta/2})$$

$$\Rightarrow \sin\frac{\pi}{11}$$

$$-i\ e^{\frac{i2\pi}{11}}\left(\frac{\cancel{\sin\frac{\pi}{11}}}{\cancel{\sin\frac{\pi}{11}}}\ e^{\frac{i9\pi}{11}}\right)$$

$$(e^{i\pi}+1=0)$$

$$(e^{i\pi}=-1)$$

$$-i\ e^{\frac{i11\pi}{11}} = -i\,e^{i\pi}$$

$$= -i\left(\cos\pi + i\sin\pi\right)$$

$$\Rightarrow -i(-1)$$

$$\Rightarrow i$$

Sumation reduces to ... $\sum_{p=1}^{32}(3p+2)\left[\ i\ \right]^{p}$

$$\Rightarrow \sum_{p=1}^{32}(3p+2)i^{p}$$

$$\Rightarrow \sum_{p=1}^{32}\left(3p\,i^{p} + 2\,i^{p}\right)$$

$$2\left(i^{1}+i^{2}+i^{3}+i^{4}+ \ldots\right)$$

$$\Rightarrow \sum_{p=1} (3p\, i^p + 2 i^p)$$

$$\Rightarrow 3\underbrace{\sum_{p=1}^{32} p \cdot i^p}_{AGP} + \cancel{2\sum_{p=1}^{32} i^p} \quad 0 \qquad \nearrow 2(i + i^2 + i^3 + i^4 + \cdots)$$

$$2 i \left(\frac{i^{32}-1}{i-1}\right) \Rightarrow 2(0) \quad 0$$

$$S = 3\{1 \cdot i^1\} + 3\{2 \cdot i^2\} \ldots\ldots + 3\{32 \cdot i^{32}\}$$

$$S = 3 \{ i + 2i^2 + 3i^3 + \cdots\cdots 32 i^{32} \}$$

AP ... $1, 2, \ldots 32$

$$iS = 3 \{ \quad i^2 + 2i^3 + \cdots\cdots 31 i^{32} + 32 i^{33} \}$$

GP $i, i^2, \ldots\ldots i^{32}$

$r = i$

$$S - iS = 3\{ i + \underbrace{i^2 + i^3 + \cdots\cdots + i^{32}}_{0} - 32 i^{33} \}$$

$$S(1-i) = 3\{ -32\, i^{33} \}$$

$$S(1-i) = -96\, \cancel{i^{32}}\, i$$

$$S(1-i) = -96 i$$

$$S = -\frac{96 i}{1-i} \times \frac{1+i}{1+i}$$

$$S = -\frac{96 i (1+i)}{1^2 + 1^2}$$

$$S = -48(i + i^2)$$

$$S = -48(-1 + i)$$

$$\boxed{S = 48(1-i)}$$

AGP

Series: $\quad a_1 \quad a_2 \quad a_3 \quad a_n$

$$\Rightarrow a, \ (a+d)r, \ (a+2d)r^2, \ \ldots\ldots (a+(n-1)d) r^{n-1}$$

$$S_n = a + (a+d)r + (a+2d)r^2 + \cdots\cdots + (a+(n-1)d) r^{n-1}$$

$$r S_n = \quad ar + (a+d)r^2 + \cdots\cdots + (a+(n-2)d) r^{n-1} + (a+(n-1)d) r^n$$

$$S_n(1-r) = a + \underbrace{dr + dr^2 + \cdots + dr^{n-1}}_{\text{GP of } n-1 \text{ terms}} - (a+(n-1)d)r^n$$

— x — x — x — x —

$$S_n(1-r) = a + \cancel{dr} \; dr\frac{(r^{n-1}-1)}{(r-1)} - (a+(n-1)d)r^n$$

$$\boxed{S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{(a+(n-1)d)r^n}{1-r}}$$

$|r| \le 1$

$n \to \infty$

special case.

$r^n \to 0$

$r^{n-1} \to 0$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$\downarrow$$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{a}{1-r} + \frac{dr(1-0)}{(1-r)^2} - \cancel{\frac{(a+(n-1)d)}{1-r}} \times 0$$

$$\boxed{S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}}$$

Ex

$1 \cdot 1, \; 3 \cdot \frac{1}{2}, \; 5 \cdot \frac{1}{4} \; \cdots$

$1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \frac{9}{16}, \frac{11}{32}, \cdots \infty$

$a = 1$

$d = 2$

$r = \frac{1}{2}$

$$S_\infty = \frac{1}{1-\frac{1}{2}} + \frac{2 \times \frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}$$

$$\rightarrow 2 + 4 = \boxed{6}$$

Q→ $1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \frac{9}{16}, \ldots\ldots$

$1, 3, 5 \ldots$

$a_n = 2n-1$

$1, \frac{1}{2}, \frac{1}{4}, \ldots$

$a_n = \frac{1}{2^{n-1}}$

$$S_n = 1 + \frac{3}{2} + \frac{5}{4} + \frac{7}{8} + \frac{9}{16} \cdots + \frac{2n-1}{2^{n-1}}$$

$$\frac{1}{2}S_n = \frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{16} \cdots + \frac{2n-3}{2^{n-1}} + \frac{2n-1}{2^n}$$

$$\frac{1}{2}S_n = 1 + \underbrace{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{2}{2^{n-1}}}_{n-1 \text{ terms}} - \frac{(2n-1)}{2^n}$$

$$\frac{S_n}{2} = 1 + \frac{1\left(\left(\frac{1}{2}\right)^{n-1} - 1\right)}{\left(\frac{1}{2} - 1\right)} - \frac{(2n-1)}{2^n}$$

$$\frac{S_n}{2} = 1 + \frac{1 - \left(\frac{1}{2}\right)^{n-1}}{\left(1 - \frac{1}{2}\right)} - \frac{(2n-1)}{2^n}$$

$$\boxed{S_n = 2 + 4\left(1 - \left(\frac{1}{2}\right)^{n-1}\right) - \frac{2n-1}{2^{n-1}}}$$

$$S_\infty = 2 + 4(1-0) - 0$$
$$\Rightarrow \underline{6}$$

More Series in Permutation & Combinations

1				2^0
1	1			$= 2^1$
1	2	1		$= 2^2$
1	3	3	1	$= 2^3$

Harmonic Progressions

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}, \ldots\ldots$$

$$\boxed{a_n = \frac{1}{a+(n-1)d}}$$

Harmonic Mean

$a, \underline{\ x\ }, b$ are in HP

$\frac{1}{a}, \frac{1}{x}, \frac{1}{b}$ are in AP

so CD is same

$$\Downarrow$$

$$\frac{1}{x} - \frac{1}{a} = \frac{1}{b} - \frac{1}{x}$$

$$\frac{2}{x} = \frac{1}{a} + \frac{1}{b}$$

—x—x—

$$\frac{2}{x} = \frac{a+b}{ab}$$

$$\boxed{x = \frac{2ab}{a+b}}$$

$AM = \frac{a+b}{2}$

$GM = \sqrt{ab}$

$HM = \frac{2ab}{a+b}$

Ex

$\frac{1}{2}, \underline{\quad}, \frac{1}{4} \Rightarrow HP$

$$\frac{2\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)+\left(\frac{1}{4}\right)} \Rightarrow \frac{\left(\frac{1}{4}\right)}{\left(\frac{3}{4}\right)} \Rightarrow \boxed{\frac{1}{3}}$$

$\boxed{a, \underbrace{x_1, x_2, \dots x_n}_{n \text{ inserted terms}}, b}$ are in HP $\quad d = ??$

$A = \frac{1}{a}$

$r+1^{th}$ term, r^{th} inserted term

$$a_{r+1} = x_r = \frac{(r+1)ab}{ra+b}$$

$$A_{n+2} = \frac{1}{b} \Rightarrow \frac{1}{a} + (n+1)d = \frac{1}{b}$$

$$\boxed{d = \frac{1}{(n+1)}\frac{(a-b)}{(ab)}}$$

Q→ Prove

A, G, H are in G.P.

or

$\underline{G^2 = AH}$

$\left(\frac{G}{A} = \frac{H}{G}\right) = r$

$$LHS = G^2 = (\sqrt{ab})^2 = ab$$

$$RHS = AH = \left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right) = ab$$

Hence proved.

Prove

a, A, G or H, b $\quad (a, b > 0)$

$$\boxed{A > G > H}$$

(1)

$A > G$

$A - G > 0$

(2)

$G > H$

$G - H > 0$

$LHS = \frac{a+b}{2} - \sqrt{ab}$

$\Rightarrow \frac{a+b-2\sqrt{ab}}{2}$

$\Rightarrow \frac{(\sqrt{a}-\sqrt{b})^2}{2} > 0$

$LHS = \sqrt{ab} - \frac{2ab}{a+b}$

$\Rightarrow \sqrt{ab}\left\{1 - \frac{2\sqrt{ab}}{a+b}\right\}$

$\Rightarrow \sqrt{ab}\left\{\frac{a+b-2\sqrt{ab}}{a+b}\right\}$

$\Rightarrow \frac{\sqrt{ab}}{a+b}\{\sqrt{a}-\sqrt{b}\}^2 > 0$ (Given $a>0$, $b>0$)

Q→ For what value of 'n' $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ ~~are in~~ ~~HP~~. is HM of a & b.

HW

AP $\boxed{n=0}$

GP $\boxed{n=-\frac{1}{2}}$

HP → $\boxed{n=-1}$

Solⁿ

a $\quad \frac{2ab}{a+b} \quad$ b

Q→ x, y, z are in GP $\quad x>1, y>1 \;\&\; z>1$

$y^2 = xz$ then $\boxed{\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}}$ are in

① AP
② GP
③ HP
④ None of these

$\boxed{\log_a a = 1}$

$\log_e x = \ln x$

$\log_e e = 1$

or $\boxed{\ln e = 1}$

$\frac{1}{\ln e + \ln x}, \frac{1}{\ln e + \ln y}, \frac{1}{\ln e + \ln z}$

$\boxed{\frac{1}{\ln(ex)}, \frac{1}{\ln(ey)}, \frac{1}{\ln(ez)}}$?? HP

$\boxed{\frac{1}{\log_a b} = \log_b a}$

$\ln ex, \ln ey, \ln ez \rightarrow$ AP

$2\ln ey = \ln ex + \ln ez$

$2\ln ey = \ln ex + \ln ez$

$\ln e^2y^2 = \ln(e^2xz)$

$y^2 = xz$

$\underline{M2}$

$x, y, z \to GP$

$ex, ey, ez \Rightarrow GP$

$\ln ex, \ln ey, \ln ez \Rightarrow AP$

$\boxed{\frac{1}{\ln(ex)}, \frac{1}{\ln(ey)}, \frac{1}{\ln(ex)} \Rightarrow HP}$

1, 2, 4 GP

↓

3, 6, 12 GP

$a, ar, ar^2, ar^3, \ldots ar^{n-1} \Rightarrow GP$

$\ln a, \ln a + \ln r, \ln a + 2\log r, \log a + 3\log r \dashrightarrow AP$

$A = \log a$

$D = \log r$

$\ln GP \Rightarrow AP$